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COMPENSATING PARASITIC HALL CURRENTS IN A MHD GENERATOR HAVING TEMPERATURE INHOMOGENEITY IN THE PLASMA FLOW

A. A. Beloglazov and B. M. Berkovskii

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The inhomogeneity parameter is examined as a function of the variation in current density over the cross section. It is pointed out that this parameter can be determined without measuring the conductivity in the cross section.

In several papers [1-8] it has been pointed out that MHD characteristics can be improved by changing the current density in the cross section of the channel by tapping off current in the magnetic-field direction. Here we introduce the inhomogeneity parameter, which incorporates not only the plasma inhomogeneity but also the inhomogeneity in the current density in the cross section, and it is used to examine the improvement in MHD characteristics. We write the expression for the Hall current by using the projections of Ohm's law on the coordinate axes and averaging them in the direction of the Y and Z axes:

$$E_x=\frac{\left(\left\langle\!\left\langle\frac{j_x}{j_y}\right\rangle\!\right\rangle+\langle\!\langle\beta\rangle\!\rangle\right)\left(\langle\!\langle E_y\rangle\!\rangle-\langle\!\langle U\rangle\!\rangle B+\frac{\langle\!\langle\beta\rangle\!\rangle\left\langle\!\left\langle\frac{j_x}{j_y}\right\rangle\!\right\rangle}{\langle\!\langle\sigma/j_y\rangle\!\rangle}\right)}{\langle\!\langle\sigma/j_y\rangle\!\rangle\left\langle\!\left\langle\frac{(1+\beta^2)\,j_y}{\sigma}\right\rangle\!\right\rangle-\langle\!\langle\beta\rangle\!\rangle^2}. \tag{1}$$

The denominator in (1) is

$$G_B=\langle\!\langle\sigma/j_y\rangle\!\rangle\left\langle\!\left\langle\frac{(1+\beta^2)\,j_y}{\sigma}\right\rangle\!\right\rangle-\langle\!\langle\beta\rangle\!\rangle^2 \tag{2}$$

and by analogy with [6, 9, 10] it is called the inhomogeneity parameter. Also, (2) contains j_y explicitly, which enables one to choose the j_y profile to minimize the inhomogeneity parameter and thus improve the MHD generator characteristics. One can influence the distribution of j_y in the cross section for example by profiling the section, and also by profiled injection, the choice of loading system, profiling the temperatures of the current-collecting surfaces, and profiling the conductivities of the electrode materials.

We use a stationary two-dimensional electrodynamic formulation to demonstrate the effects of current-density variation over the cross section on the inhomogeneity parameter.

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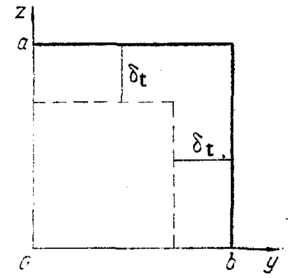

Fig. 1. Scheme for the calculation region.

We derive the distributions for the current density and electric field in a rectangular MHD channel $0 \leqslant y \leqslant a$, $0 \leqslant z \leqslant b$. We assume that the magnetic Reynolds number is small and that the external magnetic field $\bar{B} = \{0, 0, B_z\}$ is constant. We neglect the finiteness of the channel wall sectioning. The flow parameters, the channel geometry, and the boundary conditions do not vary along the channel, so

$$\frac{\partial j_x}{\partial x} = \frac{\partial E_x}{\partial x} = \frac{\partial E_y}{\partial x} = \frac{\partial E_z}{\partial x} = 0$$

and it follows additionally from rot $\bar{E} = 0$ that E_x = const. The distributions of the velocity U and stagnation enthalpy i_0 are given by the one-seventh law

$$U(y, z) = U_c\,\eta(y)\,\xi(z), \quad i(y, z) = i_w + (i_0 - i_w)\,\eta(y)\,\xi(z),$$

$$\eta(y) = \begin{cases} 1, & a - y \geqslant \delta_t \\ \left(\dfrac{a-y}{\delta_t}\right)^{1/7}, & \delta_l \leqslant a - y \leqslant \delta_t \\ \left(\dfrac{a-y}{\delta_l}\right)\left(\dfrac{\delta_l}{\delta_t}\right)^{6/7}, & a - y \leqslant \delta_l \end{cases},$$

$$\xi(z) = \begin{cases} 1, & b - z \geqslant \delta_t \\ \left(\dfrac{b-z}{\delta_t}\right)^{1/7}, & \delta_l \leqslant b - z \leqslant \delta_t \\ \left(\dfrac{b-z}{\delta_l}\right)\left(\dfrac{\delta_l}{\delta_t}\right)^{6/7}, & b - z \leqslant \delta_l \end{cases}.$$

We assume that the distributions of the gasdynamic quantities are symmetrical with respect to the axes y = 0 and z = 0 (Fig. 1). In that case, the solution can be derived for a quarter of the cross section. The treatment amounts to integrating a linear second-order elliptic equation for the potential φ:

$$-\frac{\partial}{\partial y}\left(\frac{\sigma}{1+\beta^2}\frac{\partial \varphi}{\partial y}\right) - \frac{\partial}{\partial z}\left(\sigma\frac{\partial \varphi}{\partial z}\right) = \frac{\partial}{\partial y}\left(\frac{\sigma U B}{1+\beta^2}\right) - E_x\frac{\partial}{\partial y}\left(\frac{\beta\sigma}{1+\beta^2}\right) \quad (3)$$

with the boundary conditions

$$\varphi|_{y=b} = \Phi = \text{const}, \quad \varphi|_{z=a} = \Phi y/a, \quad (4)$$

$$\left.\frac{\partial \varphi}{\partial z}\right|_{z=0} = 0, \quad \varphi|_{y=0} = 0.$$

The Hall field on the right in (3) is determined by iteration from the formula

$$E_x = \frac{\displaystyle\iint \frac{\sigma\beta}{1+\beta^2}(E_y - UB)\,dy\,dz - I_x}{\displaystyle\iint \frac{\sigma}{1+\beta^2}\,dy\,dz}. \quad (5)$$

A BÉSM-6 computer was used in solving system (3)-(5) by means of a program written at the Computing Research Center at Moscow University.

The calculations were performed with the following parameters. The velocity and temperature averaged over the cross section were U_{av} = 700 m/sec and T_{av} = 2500°K, while the pressure was P = 0.8 × 10^5 Pa, the wall temperature T_w = 2000°K, and the thicknesses of the turbulent and laminar sublayers correspondingly were δ_t = 0.13 m, δ_l = 0.0006 m. The channel was of square cross section 0.592 × 0.592, and the magnetic induction was 6 T. The working body consisted of combustion products from natural gas burned in oxygen-enriched air (40% by volume) with the addition of potassium (1% by mass). The longitudinal current I_x was taken as zero, while the load coefficient in the transverse direction was $(\Phi/a)/\langle\langle U\rangle\rangle B - 0.5$.

We can neglect the terms $\langle\langle\beta\rangle\rangle\langle\langle j_x/j_y\rangle\rangle/\langle\langle\sigma/j_y\rangle\rangle$ and $\langle\langle j_x/j_y\rangle\rangle$ in the numerator in (1) with these parameters. In fact, $\langle\langle j_x/j_y\rangle\rangle = 2.7\cdot 10^{-3}$, while $\langle\langle\beta\rangle\rangle = 4.8$ (difference about 0.04%), while $\langle\langle\beta\rangle\rangle\langle\langle j_x/j_y\rangle\rangle/\langle\langle\sigma/j_y\rangle\rangle = 22.3$ by comparison with $\langle\langle E_y\rangle\rangle - \langle\langle U\rangle\rangle B = 1948$, thus constituting about 1.1%. Then (1) agrees closely with the widely used expression derived in [10]:

$$E_x = \frac{\langle\beta\rangle(\langle E_y\rangle - \langle U\rangle B)}{\langle\sigma\rangle\left\langle\frac{1+\beta^2}{\sigma}\right\rangle - \langle\beta\rangle^2}.$$

The calculations also show that tapping off current in the magnetic-field direction, i.e., changing j_y over the cross section, substantially reduces the inhomogeneity parameter. For example, in the absence of current tapoff according to [10] it was 2.9, while with the given boundary conditions it was reduced to G_B = 1.2.

We can transform (2) by substituting for the current density:

$$G_B = \left\langle\left\langle\frac{1+\beta^2}{E_y - UB + \beta E_x}\right\rangle\right\rangle \langle\langle E_y - UB + \beta E_x\rangle\rangle - \langle\langle\beta\rangle\rangle^2. \qquad (6)$$

We see from (6) that G_B can be determined without measuring the conductivity in the working plasma volume. As it is very complicated to measure the conductivity distribution at high temperatures, it may be simpler to determine the electric fields in (6) by measuring the potential distribution in the working plasma volume. In a Faraday generator of rectangular cross section in which the current is collected only at the electrode walls, the inhomogeneity parameter of (2) or (6) takes a standard form [10] on the assumption that β = const:

$$G = \langle\sigma\rangle\left\langle\frac{1}{\sigma}\right\rangle(1+\beta^2) - \beta^2. \qquad (7)$$

For a state with weak MHD interaction, one can determine the parameter of (7) from the value of $\langle\sigma\rangle\left\langle\frac{1}{\sigma}\right\rangle$, found with the channel operating without the magnetic field but with external sources connected to the electrode walls. Then (2) and (6) with j_y = const, B = 0, β = 0 imply directly that

$$\langle\langle\sigma\rangle\rangle\left\langle\left\langle\frac{1}{\sigma}\right\rangle\right\rangle = \langle\langle E_y\rangle\rangle\left\langle\left\langle\frac{1}{E_y}\right\rangle\right\rangle. \qquad (8)$$

Therefore, one can measure the potential distribution at an insulating wall to determine $\langle E_y\rangle\left\langle\frac{1}{E_y}\right\rangle$ and therefore $\langle\sigma\rangle\left\langle\frac{1}{\sigma}\right\rangle$.

We now consider the condition for complete compensation of the parasitic Hall currents:

$$\beta j_y/\sigma = \text{const} \quad \text{or} \quad \beta(E_y - UB) = \text{const}. \qquad (9)$$

We use (9) to represent the Hall field of (1) as

$$E_x = \frac{\langle\langle\beta\rangle\rangle(\langle\langle E_y\rangle\rangle - \langle\langle U\rangle\rangle B)}{\langle\langle\beta\rangle\rangle\langle\langle 1/\beta\rangle\rangle}. \qquad (10)$$

We see from (10) that in the absence of j_{xi} the inhomogeneity parameter of (2) becomes

$$G_B = \langle\langle\beta\rangle\rangle\left\langle\left\langle\frac{1}{\beta}\right\rangle\right\rangle \qquad (11)$$

and in the limit will be equal to one for a constant Hall parameter even in the presence of current and conductivity inhomogeneities in the cross section. Then from (2) and (11) with these calculations we conclude that it is possible to minimize the inhomogeneity parameter further by profiling the current density in the cross section, for example by using joint measures to profile the current tapoff and the cross section itself. This is particularly important if lower wall temperatures are used. In this connection, it is important to measure the inhomogeneity parameter and also to perform experimental and numerical checks on these ways of improving MHD generator characteristics.

NOTATION

i_0, stagnation enthalpy; i, local enthalpy; i_w, enthalpy at channel wall; P, pressure; U, local velocity; U_{av}, velocity averaged over section; U_c, velocity at core; T_{av}, temperature averaged over section; T_w, wall temperature; $\eta(y)$, $\xi(z)$, boundary-layer profiles in the directions of the y and z axes respectively; δ_t, thickness of turbulent boundary layer; δ_l, thickness of laminar sublayer; σ, conductance; β, Hall parameter; j_x, j_y, current density projections on X and Y axes, respectively; E_x, E_y, electric field projections on x and y axes, respectively; G, inhomogeneity parameter; φ, electric potential; a, b, rectangular-region dimensions.

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